IMPULSIVE NOISE MITIGATION IN WAVELET BASED ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING SYSTEMS

BY

MAAZ ELHAG ALI

INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

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BASED ORTHOGONAL FREQUENCY DIVISION
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Multicarrier Modulation (MCM) is a widely used modulation scheme in broadband communications. It is superior to ordinary Single Carrier (SC) technique in terms of data rate, combating multipath and eliminating the effect of intersymbol interference (ISI) without using complex equalizers. The most common MCM scheme is Orthogonal Frequency Division Multiplexing (OFDM) and can be implemented using the Fast Fourier Transform (FFT) for modulation and demodulation. Wavelet-based OFDM as an alternative scheme to Fourier-OFDM has been recently adopted in some standards such as the IEEE 1901. This thesis is mainly about mitigating impulsive noise in wavelet-OFDM systems and this was achieved by using two mitigation techniques. The first method was by using blanking technique which is a common technique used in FFT-OFDM systems. This technique was used to mitigate the impulsive noise in all wavelet families under study which were Haar, Daubechies-4 and biorthogonal-4.4. The second technique, the replacing technique, was proposed and developed based on the mathematical analysis of the Haar discrete wavelet transform-OFDM. A redundancy in data was found and exploited to mitigate the impulsive noise. However, the performance of both techniques is highly dependent on the selection of threshold values. The results of the first technique showed that all wavelet types have approximately similar BER performance except the Haar wavelet family which had superior BER performance in most cases. Under the assumption of Bernoulli-Gaussian model for impulsive noise and BPSK-OFDM scheme, it was found that both techniques could achieve performance of BER of $1 \times 10^{-4}$ with 5 dB gain in SNR when the probability of impulsive noise occurrence ($p = 0.001$). Achieving higher performance requires lowering the probability of occurrence. However, Assuming that there is an improper setting of the threshold value below or near the signal level, the replacing technique showed more immunity to errors. For a Haar DWT-OFDM, BPSK modulation and probability of impulsive noise occurrence of $p = 0.01$, the replacing technique was able to improve performance by 80% above the unmitigated case while this was 30% for the blanking technique when a threshold was set below 10% of its minimum value.
ملخص البحث

تستخدم تقنية التعديل متعدد المواسم على نطاق واسع في مجال الاتصالات ذات نطاق العريض، وهي متغيرة على تقنيات الحامل الأحادي الاعتقادية من حيث زيادة معدل البيانات، ومقاومة تأثير المسار المتعدد، والتبديل من تأثير تداخل الرموز دون استخدام المعادلات المعقدة. المخطط الأكثر شيوعا من هذه التقنية هو متعامد التردد المتعدد (OFDM) ويمكن تنفيذه باستخدام توجيه فورييه السريع، وتحويل فورييه السريع العكسي (FFT/IFFT) في نظام المواسم. 

تم استخدام هذا الأساس لتطبيق تقنيات فوق-陛 ذات نطاق عريض على مجموعة متنوعة من أنواع الاتصالات. تستخدم تقنيات تخفيف الضجيج النبضي في نظام المواسم-OFDM بشكل أساسي. وتم تحقيق الهدف من خلال تقنيات تخفيف الضجيج النبضي في تقنية تعديل النطاق، التي تستخدم تقنية المسبح في نظام المواسم-OFDM. هذه الأطراف هي تقنية شائعة وفعالة مستخدمة في نظم فورييه-OFDM. وتم استخدام هذا الأساليب للتطبيق من التقنيات في جميع أنواع الاتصالات، والتي وضعها على أساس التحليل الرياضي لنظام الاتصالات المتقطعة. وتم العثور على التكرار في البيانات واستغلالها للتطبيق من الضوضاء النبضية. ومع ذلك، فإن الأداء في كل من التقنيات يعتمد بشكل كبير على اختيار قيمة العتبة (Threshold). وأظهرت نتيجة التقنية الأولى أن جميع أنواع الاتصالات المتماثلة تقريبا في أداء معدل الخطأ في الأرقام الثنائية (BER) مع تقنيات المواسم من نوع هار في أكثر الحالات. بتفاوت موجز برونلي-غادرل لضجيج النبضي، ومخطط ثنائي القطبية، فقد وجد بأنه كلا من التقنيات استطاعت تحقيق أداء في معدل خطأ في الأرقام الثنائية (BER) بمقدار 1×10^-4

مع كسب في معدل قدرة الإشارة إلى قدرة الضجيج (SNR) بمقدار 5 درجات عندما كانت احتمالية حدوث النيبضات بمقدار 0.001. لتحديد مستويات أداء أعلى يجب تخفيف احتمالية حدوث النيبضات. على كل حال، فإنه يفتراض وجود ضبط غير ملائم لقيمة العتبة بحيث تكون في مستوى الإشارة أو أدنى، فإن تقنية الاستبدال أظهرت مناحم أكثر للحيلولة. لنموذج المواسم-OFDM، مخطط ثنائي القطبية، واحتمالية حدوث نبضات بمقدار 0.01، فإن تقنية الاستبدال استطاعت تخسيس الأداء بنسبة 80% فوق حالة عدم التخفيف، بينما كانت نسبة 30% لتقنية المسح وذلك عندما كانت العتبة أدنى بمقدار 10% من قيمة الدنيا.
I certify that I have supervised and read this study and that in my opinion; it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Master of Science in Communication Engineering.

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DECLARATION

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

Maaz Elhag Ali

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Indeed, all praise is due to Allah, we praise Him, we seek His aid, and we ask for His forgiveness. We seek refuge in Allah from the evil of our actions and from the evil consequences of our actions. Whomever Allah guides, there is none to misguide and whoever Allah misguides there is none to guide. I bear witness that there is no god worthy of worship except Allah and I bear witness that Muhammad is the servant and messenger of Allah.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Multicarrier Modulation (MCM) is a widely used technique in broadband communication systems. This technique divides the transmitted data into several symbol streams of lower data rate and then transmits these substreams adjacently through subchannels after being modulated by subcarriers. The aim is to have bandwidth of each subchannel below the coherence bandwidth of the total channel, leading to flat fading subchannels (Goldsmith, 2005).

The most common scheme of MCM is Orthogonal Frequency Division Multiplexing (OFDM). Weinstein and Ebert (1971), made a major breakthrough in the implementation of multicarrier modulation. They proposed using Inverse Fourier Transform (IDFT) for modulation and Discrete Fourier Transform (DFT) for demodulation. It may be referred to as FFT-OFDM since the complexity of calculating N-DFT points can be reduced using Fast Fourier Transform (FFT).

Wavelets and filter banks are alternative methods to represent signals. They have been used in many applications like image processing and communication systems since 1980s. Unlike Fourier transform, wavelet transform uses short waves instead of long waves. When transforming to the frequency domain in Fourier transform, time information is lost. Wavelet transform was introduced to overcome this serious drawback of Fourier transform since it becomes possible to know when an event has occurred. Because of this and other properties of wavelet transform, they have been proposed in
some literature to replace FFT-OFDM systems (Strang & Nguyen, 1996).

Impulsive Noise (IN), characterized with short durations and very high amplitudes, is identified as an impairment that degrades performance of communication systems. It could be generated from man-made or atmospheric made sources. These may include switching noise, automobile engine noise, interfering electromagnetic pulses, and so on. Buses, circuits that connect the main parts of a computer, and clocks produce significant noise in laptop and desktop computers as well (Nassar, Gulati, DeYoung, Evans, & Tinsley, 2011).

1.2 PROBLEM STATEMENT AND ITS SIGNIFICANCE

Various types of noise can severely affect a wide range of communication systems. One of these noise sources is impulsive noise. For example, in a high speed Digital Subscriber Loop (DSL), impulsive noise sources may include lightning surges, vehicle ignitions, engine noise, electromagnetic discharge, transmission and switching gears (Zhidkov, 2006).

One of the studies on the effect of impulsive noise in ADSL found that without mitigating impulsive noise, impulses on ADSL lines could be $20 - 40 \, \text{dB}$ larger than either Additive White Gaussian Noise (AWGN) or near-end crosstalk; hence, a noise margin of 6 or 12 dB is not sufficient to protect ADSL from impulsive noise. Similarly, in Digital Video Broadcasting (DVB) systems, sources of impulsive noise seem to be the same as those of DSL (Al Mawali, 2011).

In Power Line Communication (PLC) systems, impulsive noise is considered the main reason for frames retransmission scenario (Zbydniewski, Zielinski, & Turcza, 29 2009-April 1). Unlike other communication environments, a channel in PLC is very
difficult to model (Al Mawali, 2011). It was found that the power spectral density (PSD) of impulsive noise is 50 dB higher than background noise (Al Mawali, 2011). In conventional OFDM systems (FFT-OFDM), it is required to add extra load, called the Cyclic Prefix (CP), to compensate for a high degree of spectral overlap.

As stated in one of the IEEE 802.16.3 proposals, overhead in wavelet-OFDM is less than of the FFT-OFDM because it does not require the addition of cyclic prefix. For wireless transmission, FFT-OFDM has a cyclic prefix of 20%; hence, wavelet-OFDM has an advantage of about 20% in bandwidth efficiency. Moreover, there is no need for pilot tones in wavelet-OFDM systems; however, some Fourier-OFDM systems use 4 out of 52 subbands for pilots which provide additional 8% advantage for wavelet-OFDM over FFT-OFDM implementations. Finally, unlike Fourier transform, wavelet transform can convert an input domain of real numbers to an output range of real numbers; hence, reducing the complexity of computation. Because wavelet-OFDM has higher spectral containment, i.e., overlapping, between sub-channels than FFT-OFDM, wavelet-OFDM is able to ameliorate the effects of narrowband interference and is more robust with respect to intersymbol interference and intercarrier interference (Zhao, Zhang, & Yuan, 2004). While most works have considered only FFT-OFDM, our work is directed to reduce the problem of impulsive noise in wavelet-OFDM.

1.3 SCOPE

This thesis focuses on wavelet-OFDM systems, particularly, the one which is described in (Abdullah, Kamarudin, Hussin, Jarrot, & Ismail, 2011) with three different wavelet families Haar, Daubechies-4 and biorthogonal-4.4. Majority of related works have considered mitigation of impulsive noise problem in FFT-OFDM systems. In this thesis,
the performance of FFT-OFDM compared with that of wavelet-OFDM in unmitigated impulsive noise environment.

Other aspects and challenges related to OFDM systems in general, like peak to average power ratio (PAPR) and frequency-timing offset are not considered in this work. The performance of a communication system is well-characterized by two curves, namely, spectral efficiency and bit error rate curves. However, this work focuses on performance in terms of bit error rate only.

1.4 RESEARCH OBJECTIVES

The following are the objectives of this work:

i. To evaluate the performance of the system in terms of bit error rate (BER) and signal to noise ratio (SNR).

ii. To compare the system performance using different wavelet families.

iii. To develop a technique capable of mitigating impulsive noise in wavelet-OFDM system.

1.5 RESEARCH METHODOLOGY

The research starts with a literature review to cover basic concepts of the topic and related works in the area. After gaining enough background of the problem under investigation, a system model to be simulated and analysed. This will be followed by a study of the effect of impulsive noise in both wavelet-OFDM and Fourier transforms based OFDM systems. Benefiting from other works in the area, a technique will be developed to mitigate impulsive noise in wavelet-OFDM systems. The methodology of this research is shown in a flow chart diagram in Figure 1.1
Literature review

System modelling and simulation (using MATLAB)

Analyse the effect of the impulsive noise in the system

Develop a technique to mitigate the impulsive noise

Study the system performance

Does the system performance meet the requirements?

Yes

Results analysis and comparison

End

No

Figure 1.1: Research methodology.
1.6 THESIS ORGANIZATION

This thesis is divided into six chapters. Chapter 1 is the introduction and contains background, problem statement, research methodology and the objectives. Chapter 2 presents a literature review, principles and related works. Selecting a communication system for simulation and taking into consideration certain assumptions, tools, environments and MATLAB implementation of some important functions used in the study are covered in Chapter 3. Chapter 4 presents a performance study of OFDM systems for both Fourier transform and wavelet transform based OFDM systems. The main objective of the thesis, mitigating impulsive noise in wavelet-OFDM systems, its performance study and comparison are presented in Chapter 5. Chapter 6 ends with the conclusion of the work, short comings and future work.
CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

A literature study is presented in this chapter covering basic concepts and principles of multicarrier modulation (MCM), practical implementation of MCM and the common techniques addressed to alleviate the effect of impulsive noise in MCM systems. The chapter is divided into seven sections. In Section 2.2, a literature review of multicarrier modulation technology is presented covering the basic principles and underlying theories of MCM systems. Section 2.3 explains the principle of Fourier based OFDM and the concept of cyclic prefix. Its counterpart, wavelet based system, is covered in Section 2.4 along with underlying theory of wavelet and filter banks. Impulsive noise definition, classes, statistical models and its effect on communication systems is presented in Section 2.5. A literature review of impulsive noise mitigation techniques is presented and evaluated in Section 2.6. Finally, Section 2.7 summarizes this chapter.

2.2 MULTICARRIER MODULATION

est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla.

Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum. (Goldsmith, 2005), (Schulze & Lüders, 2005), (Bingham, 1990) and (Hara & Prasad, 2003) Figure 2.1 shows one possible configuration of MCM. First, a bit stream of data is divided into $N$ substreams using a serial-to-parallel converter (S/P). Then, each substream is mapped into symbols $s_i = a_i + jb_i$ (e.g. QAM or PSK mapping). By using an appropriate pulse shaping, each substream of symbols is modulated via a subcarrier $f_i$. Summing all the output from each branch, the transmitted signal, $s(t)$, can be written as (Yang, 2009):

$$s(t) = \sum_{i=0}^{N-1} \Re[e^{j2\pi f_it}]$$

$$= \sum_{i=0}^{N-1} \Re[(a_i + jb_i)e^{j2\pi f_it}]$$

$$= \sum_{i=0}^{N-1} [a_i\cos(2\pi f_it) - b_i\sin(2\pi f_it)]$$

Figure 2.1: Block diagram of MCM transmission (Yang, 2009).
2.3 FOURIER BASED OFDM MODULATION

2.3.1 Cyclic Prefix

2.4 WAVELET BASED OFDM MODULATION

2.4.1 Filter Banks

Figure 2.2 shows the response of a simple lowpass filter. This can be illustrated by the following matrix operation:

\[
H_0(\omega) = \frac{1}{2} + \frac{1}{2}e^{-j\omega}.
\]

Figure 2.2: Frequency response of the lowpass filter: $H_0(\omega) = \frac{1}{2} + \frac{1}{2}e^{-j\omega}$.
\[ (\downarrow 2)x[n] = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \ldots & 1 & 0 & 0 & 0 & \ldots \\ \ldots & 0 & 1 & 0 & 0 & \ldots \\ \ldots & 0 & 0 & 0 & 1 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} \]

In general, if \( x[n] \) is an input to upsampling operation by \( L \), the output \( y[n] \) is:

\[
y[n] = \begin{cases} 
x[n/L], & n = mL \\ 0, & n \neq mL \end{cases}
\]  

(2.3)

2.4.1.1 Haar Filter Bank

\[
r_0[n] = \frac{1}{\sqrt{2}} \left( x[n] + x[n-1] \right)
\]

\[ y_0[n] = r_0[2n] \]

\[ y_0[n] = \frac{1}{\sqrt{2}} \left( x[2n] + x[2n-1] \right) \]  

(2.4)

Similarly,

\[ y_1[n] = \frac{1}{\sqrt{2}} \left( x[2n] - x[2n-1] \right) \]  

(2.5)

2.4.1.2 Perfect Reconstruction and General Structure of the Two Channel Filter Banks

\[ Q(z) = \frac{1}{2^{p-1}} \sum_{k=0}^{p-1} \binom{p+k-1}{k} (-1)^{k} z^{-(p-1)+k(1-\frac{1}{2})^{2k}} \]  

(2.6)
2.4.2 Scaling and Wavelet Functions

2.4.3 Wavelet Families

Table 2.1 shows the differences between wavelet and Fourier transforms.

Table 2.1
Some differences between wavelet and Fourier transforms (Barford et al., 1992).

<table>
<thead>
<tr>
<th>“Root” function</th>
<th>Fourier Transform</th>
<th>Wavelet Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Transform</td>
<td>( f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt )</td>
<td>( w = \int_{-\infty}^{\infty} f(t) s^{-1/2} w(t/\tau) )</td>
</tr>
<tr>
<td>Time transformed to</td>
<td>amplitude and phase for each frequency</td>
<td>amplitude for each scale and time</td>
</tr>
<tr>
<td>Input domain</td>
<td>( \mathbb{R} ) or ( \mathbb{C} )</td>
<td>( \mathbb{R} ) or ( \mathbb{C} )</td>
</tr>
<tr>
<td>Output range</td>
<td>( \mathbb{C} )</td>
<td>( \mathbb{R} ) or ( \mathbb{C} )</td>
</tr>
<tr>
<td>Localization in frequency</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Localization in time</td>
<td>No (Limited with STFT)</td>
<td>Yes</td>
</tr>
<tr>
<td>Time for fast discrete transform</td>
<td>( O(n\log n) )</td>
<td>( O(n) )</td>
</tr>
<tr>
<td>Number of non-redundant outputs of discrete transform</td>
<td>( n )</td>
<td>( n )</td>
</tr>
</tbody>
</table>

2.4.4 Different Implementation of Wavelet Based OFDM

2.4.4.1 Multiscale Wavelet Modulation (MSM)

2.4.4.2 Wavelet Pulse Shaping of PAM

2.4.4.3 Wavelet Packet Modulation (WPM)

2.4.4.4 Overlapped Discrete Wavelet Multitone Modulation (DWMT)

2.4.5 Cyclic Prefix in Wavelet Based OFDM Systems

Figure 2.3 shows the frequency responses for six subchannels for a discrete multitone (DMT), which is a Fourier based OFDM, and a discrete wavelet multitone (DWMT).

Figure 2.3b is a particular type of wavelet with \( g = 8 \), where \( g \) is the overlap factor. It is clear that DWMT has better spectral concentration than DMT.
Figure 2.3: Frequency response of six subchannels (Sandberg, 1995).
2.5 IMPULSIVE NOISE (IN)

2.5.1 Statistical Models for Impulsive Noise

2.5.1.1 Binary-State Model

2.5.1.2 Bernoulli-Gaussian Model

2.5.1.3 Poisson-Gaussian Model

2.5.1.4 Middleton Class A Model

2.5.1.5 Symmetric Alpha Stable ($\alpha$S)

2.5.2 Impulsive Noise Effect on Communication Systems

2.5.2.1 Digital Subscriber Loop (DSL)

2.5.2.2 Digital Video Broadcasting (DVB)

Table 2.2 shows DVB standard specifications (Woo et al., 2012).

<table>
<thead>
<tr>
<th></th>
<th>DVB-T</th>
<th>DVB-H</th>
<th>DVB-T2</th>
<th>DVB-S2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modulation</strong></td>
<td>QPSK, 16QAM,</td>
<td>QPSK, 16QAM,</td>
<td>QPSK, 16QAM,</td>
<td>QPSK, 8PSK,</td>
</tr>
<tr>
<td></td>
<td>64QAM</td>
<td>64QAM</td>
<td>64QAM, 256QAM</td>
<td>16APSK, 32APSK</td>
</tr>
<tr>
<td><strong>PHY FEC</strong></td>
<td>CC + RS 1/2,</td>
<td>CC + RS 1/2,</td>
<td>BCH + LDPC 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 8/9, 9/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2/3, 3/4, 7/8</td>
<td>2/3, 3/4, 7/8</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FFT Size</strong></td>
<td>2k, 8k</td>
<td>2k, 4k, 8k</td>
<td>1k, 2k, 4k, 8k, 16k, 32k</td>
<td></td>
</tr>
<tr>
<td><strong>Guard Interval</strong></td>
<td>1/4, 1/8, 1/16, 1/32</td>
<td>1/4, 1/8, 1/16, 1/32</td>
<td>1/4, 19/256, 1/8, 19/128, 1/16, 1/32, 1/128</td>
<td></td>
</tr>
<tr>
<td><strong>PHY/Link Layer I/F</strong></td>
<td>MPEG-2 TS</td>
<td>MPEG-2TS</td>
<td>BaseBand Frame</td>
<td>BaseBand Frame</td>
</tr>
<tr>
<td><strong>Link Layer</strong></td>
<td>-</td>
<td>MPE</td>
<td>GSE</td>
<td>GSE</td>
</tr>
</tbody>
</table>
2.5.2.3 Power Line Communication

Tables 2.3 shows the parameters for FFT-OFDM PHY.

<table>
<thead>
<tr>
<th>Communication method</th>
<th>Fast Fourier transform (FFT) OFDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT points</td>
<td>3072, 6144</td>
</tr>
<tr>
<td>Sampling frequency (MHz), respectively</td>
<td>75, 150</td>
</tr>
<tr>
<td>Symbol length (µs)</td>
<td>40.96</td>
</tr>
<tr>
<td>Guard interval (µs)</td>
<td>Variable according to line conditions: 5.56, 7.56, 47.12</td>
</tr>
<tr>
<td>Primary modulation (per subcarrier)</td>
<td>BPSK, QPSK, 8−, 16−, 64−, 256−, 1024−, and 4096− QAM</td>
</tr>
<tr>
<td>Frequency band (MHz)</td>
<td>2 – 30 (optional bands: 2 – 48 and 2 – 60)</td>
</tr>
<tr>
<td>Error correction</td>
<td>Turbo convolutional coding</td>
</tr>
<tr>
<td>Maximum transmission speed (Mb/s)</td>
<td>545 (8/9 CTC)</td>
</tr>
<tr>
<td>Diversity modes</td>
<td>Normal ROBO, mini ROBO, high-speed ROBO, and frame control</td>
</tr>
</tbody>
</table>

2.5.2.4 Wireless Communications

2.6 IMPULSIVE NOISE MITIGATION TECHNIQUES

2.7 SUMMARY
CHAPTER 3
SYSTEM MODELLING

3.1 INTRODUCTION

3.2 OFDM SYSTEM

3.2.1 Wavelet-Based OFDM System

\[ s = \text{conv}(\text{dyadup}(x_a), f_0) + \text{conv}(\text{dyadup}(x_d), f_1); \]

while the analysis side (receiver) is implemented as:

\[ x_a = \text{dyaddown}(\text{conv}(s, h_0)) \]
\[ x_d = \text{dyaddown}(\text{conv}(s, h_1)) \]

where the command ‘conv’ is a convolution operation, ‘dyadup’ and ‘dyaddown’ are, respectively, upsampling and downsampling (by 2) operations, \(f_0\) and \(f_1\) are the lowpass and the highpass filter coefficients, respectively, of the synthesis side; \(h_0\) and \(h_1\) are the lowpass and highpass filter coefficients of the analysis side. All these terms are discussed in Section 2.4.

3.2.1.1 Haar

3.2.1.2 Daubechies-4 (Db2)

3.2.1.3 Biorthogonal-4.4 (Bior4.4)

Figure 3.1 shows properties of this wavelet.
Figure 3.1: Properties of ‘bior4.4’ wavelet.
CHAPTER 4

PERFORMANCE OF THE OFDM SYSTEMS OVER AWGN & IMPULSIVE NOISE

4.1 INTRODUCTION

4.2 PERFORMANCE OF OFDM SYSTEMS OVER AWGN

4.2.1 Analysing the Performance of OFDM Systems

4.2.2 Considerations for Signal Energy Calculations in OFDM Systems

4.2.2.1 Signal Energy Calculations in FFT-OFDM

4.2.2.2 Signal Energy Calculations in DWT-OFDM

4.2.3 Monte Carlo Simulation

4.2.3.1 Procedures of Monte Carlo Simulation

4.2.4 Simulation Results and Analysis

Table 4.1
Results of OFDM systems in AWGN channel.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Required SNR to achieve</th>
<th>SER = 10^{-5}</th>
<th>BER = 10^{-5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>9.5 dB</td>
<td>9.5 dB</td>
<td></td>
</tr>
<tr>
<td>QPSK</td>
<td>13 dB</td>
<td>12.5 dB</td>
<td></td>
</tr>
<tr>
<td>16-QAM</td>
<td>20 dB</td>
<td>19.3 dB</td>
<td></td>
</tr>
</tbody>
</table>
Procedure 4.1 Monte Carlo Simulation (FFT-OFDM)

procedure CALCULATEBitErrorRate (BER)

\[ N_{FFT} = 64, N_{st} = 52, N_{cp} = 16 \]
\[ n\text{Symbols} = 5000; \]
\[ n\text{Bits} = n\text{Bit perSymbol} \times n\text{Symbol}, \]
\[ \text{SNR} = [0 : 2 : 20], \]
\[ \text{SNR}_{FFT,eff} = \text{SNR} + 10\log_{10}(\frac{N_{st}}{N_{FFT}}) + 10\log_{10}(\frac{N_{FFT}}{N_{cp}+N_{FFT}}) \]

for \( i = 1 : \text{length(SNR)} \) do

\[ \text{ipBits} \mapsto X_i \]
\[ x_n = \text{ifft}(X_i) \]
\[ s_k = x_n + x_n(49 : 64) \]
\[ w_k = \frac{1}{\sqrt{2}}(\text{randn}(1,\text{length}(s_k)) + j \times \text{randn}(1,\text{length}(s_k))) \]
\[ r_k = s_k + 10^{-\text{SNR}_{eff} \times 10} \times w_k \]
\[ y_n = r_k(17 : 80) \]
\[ Y_i = \text{fft}(y_n) \]
\[ n\text{SymbolsError} \leftarrow Y_i \neq X_i \]
\[ Y_i \mapsto \text{opBits} \]
\[ n\text{BitsError} \leftarrow \text{ipBits} \neq \text{opBits} \]

end for

\[ \text{SER} = \frac{n\text{SymbolsError}}{n\text{Symbols} \times N_{st}} \]
\[ \text{BER} = \frac{n\text{BitsError}}{n\text{Bits}} \]
end procedure

4.3 PERFORMANCE OF THE OFDM SYSTEMS OVER BOTH AWGN & IN CHANNEL

4.3.1 Varying Both SNR and SINR \( (\sigma_g^2 = f \sigma_w^2) \)

4.3.2 Fixing SNR Varying SINR

4.3.3 Fixing SINR Varying SNR

4.3.4 Simulation Results and Analysis

4.3.4.1 Results Obtained By Maintaining: \( \sigma_g^2 = f \sigma_w^2 \)

Figures 4.1, 4.2 and 4.3 show the results obtained accordingly.

From Figure 4.1a, for example, ‘heavily-disturbed environment’, \( p = 0.1 \), and high impulsive noise power \( (\sigma_g^2 = 10\sigma_w^2) \), it can be seen it is reduced in cases of increasing the impulsive power; for example, when \( (\sigma_g^2 = 100\sigma_w^2) \) (Figure 4.2a).FFT-OFDM
curve moves closer to the AWGN when decreasing the probability of occurrence, for example, it has the same performance for AWGN to achieve $10^{-4}$ (Figure 4.1c). (Figures (4.1c) and 4.2c). DWT-OFDM is superior in performance in regions of low BER. The best performance of DWT-OFDM was achieved in the simulation compared to FFT is the situation when very high IN power in an environment weakly-disturbed by IN; for example, 4.3c where at SNR$= 10$ dB DWT could achieve BER of $1 \times 10^{-3}$, while it is $1 \times 10^{-2}$ for FFT-OFDM. Table 4.2 summarizes theses results.
Figure 4.1: Performance of BPSK OFDM systems in AWGN & IN ($\sigma_g^2 = 10\sigma_w^2$ with different values of occurrence probability, $p$).
Figure 4.2: Performance of BPSK OFDM systems in AWGN & IN ($\sigma_g^2 = 100\sigma_w^2$ with different values of occurrence probability $p$).
Figure 4.3: Performance of BPSK OFDM systems in AWGN & IN ($\sigma_g^2 = 1000\sigma_w^2$ with different values of occurrence probability, $p$).
Table 4.2
BPSK OFDM performance under AWGN & IN, $\sigma_g^2 = f \sigma_w^2$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\sigma_g^2 = 10 \sigma_w^2$</th>
<th>$\sigma_g^2 = 100 \sigma_w^2$</th>
<th>$\sigma_g^2 = 1000 \sigma_w^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AWGN (BER of DWT)</td>
<td>DWT (BER of DWT)</td>
<td>FFT (BER of DWT)</td>
</tr>
<tr>
<td>0.1</td>
<td>$8.5 , \text{dB}$</td>
<td>$15.8 , \text{dB}$ $1 \times 10^{-2}$</td>
<td>$12$ $3 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.01</td>
<td>$8.5 , \text{dB}$ $1.5 \times 10^{-3}$</td>
<td>$13 , \text{dB}$</td>
<td>$8.8 , \text{dB}$ $1 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.001</td>
<td>$8.5 , \text{dB}$ $2 \times 10^{-4}$</td>
<td>$9.5 , \text{dB}$</td>
<td>$8.5 , \text{dB}$ $2 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$8.5 , \text{dB}$ $6 \times 10^{-2}$</td>
<td>$25 , \text{dB}$ $7 \times 10^{-3}$</td>
<td>$19$ $7 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.01</td>
<td>$8.5 , \text{dB}$ $7 \times 10^{-3}$</td>
<td>$22.5 , \text{dB}$</td>
<td>$13.5 , \text{dB}$ $3 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.001</td>
<td>$8.5 , \text{dB}$ $8 \times 10^{-4}$</td>
<td>$18 , \text{dB}$ $5 \times 10^{-4}$</td>
<td>$11 , \text{dB}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$8.5 , \text{dB}$ $7 \times 10^{-2}$</td>
<td>$35 , \text{dB}$</td>
<td>$29.5 , \text{dB}$ $7 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.01</td>
<td>$8.5 , \text{dB}$ $8 \times 10^{-3}$</td>
<td>$33 , \text{dB}$ $4 \times 10^{-3}$</td>
<td>$22.5 , \text{dB}$</td>
</tr>
<tr>
<td>0.001</td>
<td>$8.5 , \text{dB}$ $8 \times 10^{-4}$</td>
<td>$28 , \text{dB}$ $6 \times 10^{-4}$</td>
<td>$18.5 , \text{dB}$</td>
</tr>
</tbody>
</table>
CHAPTER 5

MITIGATION OF IMPULSIVE NOISE IN DWT-BASED OFDM SYSTEMS

5.1 INTRODUCTION

5.2 MITIGATION OF IMPULSIVE NOISE USING BLANKING TECHNIQUE

5.2.1 Threshold Selection

5.2.1.1 Fixed Threshold

5.2.1.2 Optimized Threshold

Algorithm 5.1 Finding the optimum threshold

procedure FINDOPTIMUMTHRESHOLD ($T_{th, opt}$)

SNR = [0 : 2 : 20], $T_{th} = [0 : 0.1 : 7]$

for i=1:length(SNR) do

    for j=1:length($T_{th}$) do

        procedure CALCULATEBITERRORRATE ($BER(T_{th,j})$)

    end procedure

    $T_{opt,i} = T_{th}(j) \leftarrow \text{minimum}(BER(T_{th,j}))$

end for

end procedure
5.2.1.3  *Expected Threshold*

5.2.2  Simulation Results and Analysis

5.2.2.1  *Results of Fixed Threshold*

5.2.2.2  *Results of Optimized Threshold*

5.2.2.3  *Results of Expected Threshold*

5.3  IMPULSIVE NOISE MITIGATION USING THE REPLACING TECHNIQUE FOR HAAR DWT BASED OFDM

5.3.1  Simulation Results and Analysis

5.4  PERFORMANCE COMPARISON

5.5  SUMMARY
CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 CONCLUSION

6.2 KEY FINDINGS, CONTRIBUTIONS AND SHORT COMINGS

6.3 FUTURE WORK
REFERENCES


APPENDIX A

BIT ERROR RATE (BER) FOR BPSK MODULATION

In Binary phase shift keying scheme, the input bits 1 and 0 could be represented by two analogue levels; $\sqrt{E_b}$ for symbol 1 and $-\sqrt{E_b}$ for symbol 0. Figure A.1 shows a block diagram for a typical BPSK transmitter and receiver.

When the signal is to be sent over the channel it will experience noise $n$, which is additive white Gaussian noise.

The received signal:

$$ y = \begin{cases} 
    s_1 + n, & \text{if bit 1 is transmitted} \\
    s_0 + n, & \text{if bit 0 is transmitted} 
\end{cases} $$

The conditional probability distribution function (PDF) (Figure A.2) of $y$ for the two signals are:

$$ f(y/s_0) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y+\sqrt{E_b})^2}{N_0}} \quad (A.1) $$

$$ f(y/s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y-\sqrt{E_b})^2}{N_0}} \quad (A.2) $$
Figure A.2: The conditional probability distribution function with BPSK.

Assuming an equiprobable transmitted bits, i.e, \( P(s_1) = P(s_0) = \frac{1}{2} \), the threshold 0 is the optimal decision boundary. If the received signal is greater than the threshold, the receiver decides \( s_1 \) was transmitted, otherwise, it decides that \( s_0 \) is transmitted. Mathematically:

\[
\begin{align*}
    y > 0 & \Rightarrow s_1 \\
    y \leq 0 & \Rightarrow s_0
\end{align*}
\]
The probability of error given $s_1$ was transmitted:

$$f(e|s_1) = \frac{1}{\sqrt{\pi} N_0} \int_{-\infty}^{0} e^{-\frac{(y-\sqrt{E_b})^2}{N_0}} dy \quad (A.3)$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz \quad (A.4)$$

$$= \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (A.5)$$

where

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-x^2} dx \quad (A.6)$$

is the complementary error function.

Similarly, the probability of error given $s_0$ is transmitted

$$f(e|s_0) = \frac{1}{\sqrt{\pi} N_0} \int_{0}^{\infty} e^{-\frac{(y+\sqrt{E_b})^2}{N_0}} dy \quad (A.7)$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz \quad (A.8)$$

$$= \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (A.9)$$

The total probability of bit error:

$$P_b = P(s_1)f(e|s_1) + P(s_0)f(e|s_0) \quad (A.10)$$

$$P_b = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (A.11)$$
APPENDIX B

SYMBOL ERROR RATE (SER) FOR QPSK MODULATION

\[ \sqrt{\frac{E_s}{2}}(-1 + j) \quad s_3 \circ \]
\[ \sqrt{\frac{E_s}{2}}(+1 + j) \quad s_2 \circ \]
\[ \sqrt{\frac{E_s}{2}}(-1 - j) \quad s_0 \circ \]
\[ \sqrt{\frac{E_s}{2}}(1 - j) \quad s_1 \circ \]

Figure B.1: QPSK constellation.

Assuming the alphabets used for QPSK are \( \alpha_{QPSK} = \{\pm 1\pm j\} \), the constellation diagram for QPSK is shown in Figure B.1. The factor \( \sqrt{\frac{E_s}{2}} \) is a normalization factor, i.e., to normalize the average energy of the transmitted symbols to 1, assuming that all symbols are equiprobable.

The conditional probability distribution function (PDF) of \( y \) given \( s_2 \) was transmitted (Figure B.2):

\[
f(y/s_2) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y-\sqrt{E_s})^2}{2N_0}} \quad (B.1)
\]

From figure B.2, symbol \( s_2 \) is detected correctly if \( y \) falls in the hashed area, i.e.,

\[
f(c|s_2) = f(\Re\{y\} > 0|s_2)f(\Im\{y\}|s_2) \quad (B.2)
\]
Probability of real part of $y > 0$, given $s_2$ was sent (i.e., dark area) is:

\[
f(\Re\{y\} > 0 | s_2) = 1 - \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\frac{(\Re\{y\} - \sqrt{E_s})^2}{N_0}} \, dy \quad (B.3)
\]

\[
= 1 - \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) \quad (B.4)
\]

Similarly, for the imaginary part of $y$ (Figure B.3):

\[
f(\Im\{y\} > 0 | s_2) = 1 - \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\frac{(\Im\{y\} - \sqrt{E_s})^2}{N_0}} \, dy \quad (B.5)
\]

\[
= 1 - \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) \quad (B.6)
\]
The probability of $s_2$ being decoded correctly is,

$$f(c|s_2) = \left[ 1 - \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) \right]^2$$  \hspace{1cm} (B.7)

$$= \left[ 1 - \frac{2}{2} \operatorname{erfc}\left(\sqrt{E_s}\right) + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{E_s}\right) \right]$$  \hspace{1cm} (B.8)

$$= 1 - \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{E_s}\right)$$  \hspace{1cm} (B.9)

$$P_{e,QPSK} = 1 - f(c|s_2)$$  \hspace{1cm} (B.10)

$$= 1 - \left[ 1 - \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{E_s}\right) \right]$$  \hspace{1cm} (B.11)

$$= \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) - \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{E_s}{2N_0}}\right)$$  \hspace{1cm} (B.12)

For higher values of $E_s/N_0$, the approximated equation is:

$$P_{e,QPSK} \approx \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)$$  \hspace{1cm} (B.13)
APPENDIX C

SYMBOL ERROR RATE (SER) FOR 16-QAM MODULATION

Figure C.1: 16-QAM constellation.

The average signal energy is given by:

\[
E_{\text{avg}} = \sum_{m=1}^{M} p_mE_m
\]  

(C.1)
where \( pm \) is the message probability. \( E_m \) is symbol energy for message \( m \), \( M \) is the modulation order, for 16-QAM, \( M = 16 \). If all symbols have the same probability (e.g. equiprobable message) the equation (C.1) becomes:

\[
E_{\text{avg}} = \frac{5}{2} d^2 \quad (C.2)
\]

where \( d \) is the minimum distance between two consecutive constellations. In terms of average energy per bit \( E_{b\text{avg}} \):

\[
E_{b\text{avg}} = \frac{E_{\text{avg}}}{\log_2 M} \quad (C.3)
\]

\[
d = \frac{\sqrt{E_{\text{avg}}}}{2} \quad (C.4)
\]

The alphabets of a 16-QAM modulation scheme are:

\[
\alpha = \{ \pm 1 + \pm 1j, \pm + \pm 3j, \pm + \pm 3j, \pm + \pm 1j \}
\]

The conditional probability distribution function (PDF) of \( y \) given \( s_5 \) was transmitted:

\[
f(y|s_5) = \frac{1}{\sqrt{\pi N_0}} e^{-\left(\frac{y - \sqrt{E_{\text{avg}}}}{\sqrt{N_0}}\right)^2} \quad (C.5)
\]

From figure C.1, symbol \( s_5 \) is detected correctly if \( y \) falls in the horizontally shaded area, i.e.,

\[
f(c|s_5) = f \left( \Re\{y\} \leq 0, \Re\{y\} < -2 \sqrt{\frac{E_{\text{avg}}}{10}} | s_5 \right) = f \left( \Im\{y\} > 0, \Im\{y\} < 2 \sqrt{\frac{E_{\text{avg}}}{10}} | s_5 \right).
\]
Using the relation:

\[ f(c|s_5) = \left[ 1 - \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \left[ 1 - \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \quad (C.6) \]

The probability of error for \( s_5 \) is:

\[ f(e|s_5) = 1 - \left[ 1 - \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right]^2 \quad (C.7) \]

\[ \approx 2\text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \quad (C.8) \]

The conditional probability distribution function (PDF) of \( y \) given \( s_3 \) was transmitted is:

\[ f(y|s_3) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y - \sqrt{\frac{E_{\text{avg}}}{10}})^2}{N_0}} \quad (C.9) \]

From Figure 6, symbol \( s_3 \) is detected correctly if \( y \) falls in the crossed shaded area, i.e.,

\[ f(c|s_3) = f \left( \Re \{y\} > 2 \sqrt{\frac{E_{\text{avg}}}{10}} | s_3 \right) f \left( \Im \{y\} > 2 \sqrt{\frac{E_{\text{avg}}}{10}} | s_3 \right) \]

Using the relation:

\[ f(c|s_3) = \left[ 1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \left[ 1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \quad (C.10) \]
The probability of error for $s_3$ is:

\[
f(e|s_3) = 1 - \left[ 1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right]^2 \quad (C.11)
\]

\[
\approx \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \quad (C.12)
\]

The conditional probability distribution function (PDF) of $y$ given $s_{11}$ was transmitted:

\[
f(y|s_{11}) = \frac{1}{\sqrt{\pi N_0}} e^{-\left(\frac{y - \sqrt{E_{\text{avg}}}}{N_0}\right)^2} \quad (C.13)
\]

From figure C.1, symbol $s_{11}$ is detected correctly if $y$ falls in the vertically shaded area, i.e.,

\[
f(c|s_{11}) = f \left( \Re \{y\} > 2\sqrt{\frac{E_{\text{avg}}}{10}} \mid s_{11} \right) \cdot f \left( \Im \{y\} \leq 0, \Im \{y\} > -2\sqrt{\frac{E_{\text{avg}}}{10}} \mid s_{11} \right)
\]

Using the above two cases as reference

\[
f(c|s_{11}) = \left[ 1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \left[ 1 - \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \quad (C.14)
\]

The probability of error of $s_{11}$:

\[
f(e|s_{11}) = 1 - \left[ 1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \left[ 1 - \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \right] \quad (C.15)
\]

\[
\approx \frac{3}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \quad (C.16)
\]
The total symbol probability of 16-QAM:

\[ P_{e,16QAM} \approx \frac{3}{2} \text{erfc} \left( \sqrt{\frac{E_{\text{avg}}}{10N_0}} \right) \]  

(C.17)